ON THE APPROXIMATE REALIZATION OF RESPONSES USING TRANSIENT CURVES

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The problem of realizing the transient response is often solved separately from the problem of realizing a given trajectory. This means that if the initial condition does not coincide with the initial position of the trajectory being realized, then, at first the transient response which permits us to hit onto the trajectory is produced by one method and then the problem of realizing this trajectory is solved by another method (see, for example, [1]). Here we shall consider a method, using the results of Barbashin [2-4] on the approximate realization of a trajectory, which allows us to solve these problems by a single method. The main point of the proposed method consists of the following. It is considered that by some means there is given a family of transient curves determining a direction field in the phase coordinate space. The given system of differential equations also determines some direction field which depends on control functions. The control functions are then found from the conditions of minimization at each instant of time of the square of the deviation between corresponding vectors from the two above-mentioned direction fields.

1. We shall consider the system of differential equations

$$\frac{dx_i}{dt} = \sum_{k=1}^{n} a_{ik}(t)\, x_k + \sum_{k=1}^{m} b_{ik} u_k \qquad (i = 1, \ldots, n;\ \ m \leqslant n) \tag{1.1}$$

Here b_{ik} are constants, u_k are scalar control functions which may depend both on time and on the phase coordinates. In matrix form system (1.1) can be written in the following way:

$$dx \,/\, dt = A(t)\, x + Bu \tag{1.2}$$

where $A(t)$ is an nth order square matrix, B is a rectangular matrix of order $n \times m$. We shall assume that the rank of B is m, or, equivalently, that the vectors $b_k(b_{1k}, \ldots, b_{nk})$, forming the columns of matrix B, are linearly independent.

When $t_0 \leqslant t < \infty$, let there be given the curves $x_i = \psi_i(t)$, $(i = 1, \ldots, n)$, or, in vector form $x = \psi(t)$. In the phase coordinate space we shall consider that there is given somehow a certain n-parameter family of curves

$$f(x_0, \tau, t) \tag{1.3}$$

Here, the coordinates of vector x_0 themselves represent n of the parameters, $t_0 \leqslant \tau \leqslant t < \infty$, and the time τ is determined from the condition that each curve of family (1.3) is found to be at the point x_0 at the instant τ, i.e.

$$f(x_0, \tau, \tau) = x_0 \tag{1.4}$$

The curves from family (1.3) will be called transients. It will be assumed that $x = \psi(t)$ belongs to family (1.3). It is then understood that $f(\psi(\tau), \tau, t) = \psi(t)$. For any fixed x_0 and τ we shall consider the function $f(x_0, \tau, t)$ to be piecewise-continuously differentiable with respect to t. Then, under very broad conditions, family (1.3) can be considered as the solution of a certain system of differential equations

$$\frac{df}{dt} = F(f, t) \qquad (t_0 \leqslant t < \infty) \tag{1.5}$$

where, in general, $F(f, t)$ is a piecewise-continuous function of t. In what follows it is natural to choose a family $f(x_0, \tau, t)$ such that each curve in it infinitely approximates to the curve $\psi(t)$ as $t \to \infty$. In order to fulfill this condition the solution $\psi(t)$ of system (1.5) should be globally asymptotically stable.

If the initial point $x_0 = x(t_0)$ of system (1.2) does not coincide with the point $\psi(t_0)$, then we are faced with the problems of realizing the transient response and the given response $\psi(t)$. We shall solve these problems by a single method, using the transient curves of family (1.3) which has been introduced.

For this we can select control functions $u_i(t)$ such that the solution $x(t)$ of system (1.2), determined by the initial condition $x(t_0) = x_0$, will be some approximation of the transient curve $f(x_0, t_0, t)$ from family (1.3). Since the initial point of this curve and the initial position of system (1.2) coincide, then, the control functions can be selected as was done in [2,3]. In [2] it was proved that the control functions are determined from the system of equations

$$\sum_{i=1}^{m} (b_k, b_i)\, u_i(t) = (r(t), b_k) \qquad (k = 1, \ldots, m) \tag{1.6}$$

where (b_k, b_i) is the scalar product of vectors b_k and b_i, and the vector function is

$$r(t) = f_t'(x_0, t_0, t) - A(t)\, f$$

However, if the number of control functions $m < n$, then after as small as desired interval of time Δt, the position $x(t_0 + \Delta t)$ determined by system (1.2), in general, will not coincide with the point $f(x_0, t_0, t_0 + \Delta t)$. However, the transient curve $f(x(t_0 + \Delta t), t_0 + \Delta t, t)$ of family (1.3) will pass through the point $x(t_0 + \Delta t)$. Therefore, at the instant $t_0 + \Delta t$ it is natural to solve the problem of realizing the trajectory $f(x(t_0 + \Delta t), t_0 + \Delta t, t)$, and, in correspondence with this, to find the control functions $u_i(t)$ from Equation (1.7). By proceeding in this manner for each interval of time Δt, we shall find the control function $u(t)$ from equations of form (1.6) by substituting the corresponding transient curve from family (1.3) for the vector function $r(t)$.

We shall assume that the time interval Δt is infinitesimal. At the instant τ let the trajectory of system (1.2) pass through the point $x(\tau)$. Then, in order to find control $u(\tau)$ at each instant of time τ, we should solve the problem of realizing the transient curve $f(x(\tau), \tau, t)$. In this case, control u is found as the function $u(x(\tau), \tau)$ from the following system of linear equations:

$$\sum_{i=1}^{m} (b_k, b_i)\, u_i(x(\tau), \tau) = (r(x(\tau), \tau), b_k) \tag{1.7}$$

Here

$$r(x(\tau), \tau) = \frac{\partial f(x(\tau), \tau, t)}{\partial t}\bigg|_{t=\tau} - A(\tau)\, f(x(\tau), \tau, \tau)$$

or, by virtue of (1.4), it follows that

$$r(x(\tau), \tau) = \frac{\partial f(x(\tau), \tau, t)}{\partial t}\bigg|_{t=\tau} - A(\tau)\, x(\tau) \tag{1.8}$$

By taking into account that $x(\tau)$ is the solution of system (1.2), we can consider that the control obtained from Equation (1.7) is a function $u(x, \tau)$ of the phase coordinates and of time. Formally, $u(x, \tau)$ can be obtained from Equation (1.7) by reckoning that $x(\tau)$ does not depend on τ. The very same control $u(x, \tau)$ can be obtained from the condition of best square approximation, at each instant of time τ, of the velocity vector of trajectory $f(x, \tau, t)$ by the velocity vector of the solution of system (1.2). In fact, in this case we should select the control function $u(x, \tau)$ such that the quantity

$$\left\| A(\tau)\, x + Bu - \frac{\partial f(x, \tau, t)}{\partial t}\Big|_{t=\tau} \right\| \qquad \left(\|x\| = \Big[\sum_{i=1}^{n} x_i^2\Big]^{1/2}\right) \tag{1.9}$$

is minimized.

By the same reasoning as in [2], according to [5, p. 205], it is not difficult to prove that the desired control $u(x, \tau)$ should satisfy system (1.7). If the family of transient curves (1.3) is given as the solution of system (1.5), then

$$\frac{\partial f(x, \tau, t)}{\partial t}\Big|_{t=\tau} = F(x, \tau)$$

and system (1.7) is rewritten as

$$\sum_{i=1}^{m} (b_k, b_i)\, u_i(x, \tau) = (F(x, \tau) - A(\tau)\, x, b_k) \qquad (k = 1, \ldots, m) \tag{1.10}$$

Formulas (1.7) and (1.10) are simple when the system of vectors $b_1, \ldots, b_m$ is orthonormal. Then

$$u_i(x, t) = (r(x, t), b_i) \tag{1.11}$$

Remark 1. It was assumed above that the given curve $\psi(t)$ belongs to the family of transient curves. It is not always natural to impose such a condition on the choice of family (1.3). It can be considered that the transient curves will reach curve $\psi(t)$ in finite intervals of time, and, in this connection, the derivatives of the transient curve and of curve $\psi(t)$ do not coincide at the instant of contact. In this case if the trajectory of system (1.2) by virtue of controls found from Equations (1.7), will reach curve $\psi(t)$ in a finite interval of time, and if we continue to make use of these controls further, then, in practice, we obtain a sliding state.

Remark 2. Let there be given a nonlinear system of differential equations

$$\frac{dx}{dt} = \Phi(x, t) + Bu \tag{1.12}$$

the curve $\psi(t)$, and the family of transient curves (1.3). We shall select $u(x, \tau$ such that the quantity

$$\left\| \Phi(x,\ \tau) + Bu - \frac{\partial f(x,\ \tau,\ t)}{\partial t}\bigg|_{t=\tau} \right\|$$

which has the same meaning as quantity (1.9), is minimized at each instant of time. Then, for example, if the system of vectors $b_1,\ \ldots,\ b_m$ is orthonormal, and if family (1.3) is given as the solution of the system of differential equations (1.5), we shall have

$$u_k(x,\ \tau) = (F(x,\ \tau) - \Phi(x,\ \tau),\ b_k)$$

Let us consider an example. It will be assumed that curve $\psi(t)$ and system (1.2) are given. The family of transient curves is obtained as the curves of pursuit for curve $\psi(t)$. By assuming that the absolute value of the velocity of the pursuit curve is known, and is some scalar function $v(t)$, we find the system of differential equations which is satisfied by the pursuit curve. From Fig. 1 it is obvious that

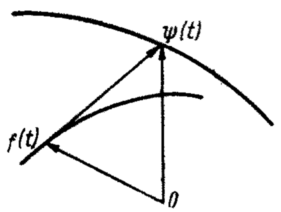

Fig. 1.

$$\frac{\psi(t) - f(t)}{\|\psi(t) - f(t)\|} = \frac{f'(t)}{v(t)}$$

or

$$\frac{df}{dt} = \frac{v(t)}{\|\psi(t) - f(t)\|}[\psi(t) - f(t)]$$

For simplicity we assume that the system of vectors $b_1,\ \ldots,\ b_m$ is orthonormal. Then, the controls are found by Formulas (1.11)

$$u_i(x,\ t) = \left(\frac{v(t)}{\|\psi(t) - x\|}(\psi(t) - x) - A(t)\,x,\ b_i\right)$$

2. Let us consider in more detail the case when the family of transient curves $f(x_0,\ t_0,\ t)$ is given as the solution of a certain system of linear differential equations

$$df\,/\,dt = Cf - C\psi + d\psi\,/\,dt \tag{2.1}$$

Here C is a square matrix which, in general, depends on time. It is clear that $\psi(t)$ is a solution of this system. If matrix C is such that the homogeneous system corresponding to system (2.1) is asymptotically stable with respect to the origin, then $\psi(t)$ is an asymptotically stable motion of system (2.1). It is not difficult to find the control function for system (1.2) from Equations (1.10) by substituting into them the right-hand side of system (2.1) instead of $F(x,\ t)$. If the system of vectors $b_1,\ \ldots,\ b_m$ is orthonormal (for simplicity in what follows this will be assumed to be the case), then the control functions are found in the form

$$u_i(x, t) = \left(C(x - \psi) + \frac{d\psi}{dt} - Ax, b_i\right) \qquad (i = 1, \ldots, m) \tag{2.2}$$

If in Equation (2.1), the matrix $A(t)$ is chosen as matrix $C(t)$, then controls (2.2) (and also those found from Equations (1.10)) will be identical to the controls obtained in [2]. Thus, in the particular case when $C(t) \equiv A(t)$, the controls found in [2] approximate the curve $\psi(t)$ by the family of transient curves which are the solutions of the linear system of differential equations (2.1). By substituting Equation (2.2) into system (1.1), we obtain the following system of differential equations:

$$\frac{dx_i}{dt} = \sum_{k=1}^{n} a_{ik}(t)\, x_k + \sum_{k=1}^{m} ([C - A]\, x, b_k)\, b_{ik} + \sum_{k=1}^{m} \left(\frac{d\psi}{dt} - C\psi, b_k\right) b_{ik} \qquad (i = 1, \ldots, n) \tag{2.3}$$

In system (2.3) we make the change of variables $z = x - \psi(t)$. Then we have the new system of equations

$$\frac{dz_i}{dt} = \sum_{k=1}^{n} a_{ik}(t)\, z_k + \sum_{k=1}^{m} ([C - A]\, z, b_k)\, b_{ik} + \\ + \sum_{k=1}^{n} a_{ik}(t)\, \psi_k(t) - \frac{d\psi_i}{dt} + \sum_{k=1}^{m} \left(\frac{d\psi}{dt} - A\psi, b_k\right) b_{ik} \tag{2.4}$$

which can be rewritten in the matrix form

$$\frac{dz}{dt} = [A + D(C - A)]\, z + A\psi - \frac{d\psi}{dt} + \sum_{i=1}^{m} \left(\frac{d\psi}{dt} - A\psi, b_i\right) b_i \tag{2.5}$$

Here

$$D = \sum_{i=1}^{m} D_i, \qquad D_i = \left\| \begin{matrix} b_{i1}^2 & b_{i1}b_{i2} & \ldots & b_{i1}b_{in} \\ b_{i2}b_{i1} & b_{i2}^2 & \ldots & b_{i2}b_{in} \\ \ldots & \ldots & \ldots & \ldots \\ b_{in}b_{i1} & b_{in}b_{i2} & \ldots & b_{in}^2 \end{matrix} \right\|$$

Let $z_0 = x_0 - \psi(t_0)$. Then the solution $z(t)$ of Equation (2.5) can be found by the Cauchy formula

$$z(t) = F(t)\, z_0 + \int_{t_0}^{t} F(t)\, F^{-1}(\tau)\, y(\tau)\, d\tau \tag{2.6}$$

Here $F(t)$ is the fundamental matrix of the solutions of the homogeneous system of equations

$$\frac{dz}{dt} = [A + D\ (C - A)]\ z = Hz \qquad (H = A + D\ (C - A)) \tag{2.7}$$

which corresponds to system (2.5), and the vector-function $y(\tau)$ has the form

$$y(\tau) = A\psi - \frac{d\psi}{dt} + \sum_{i=1}^{m} \left(\frac{d\psi}{dt} - A\psi,\ b_i\right) b_i$$

Let us clarify the possibility of reducing the deviation $z(t)$. Let us note that the vector-function $y(t)$ does not depend on the choice of family prescribed in the form (2.1) (i.e. in the final analysis, does not depend on the choice of matrix C), and it, consequently, can be obtained if in Equation (2.1) we use matrix A for matrix C. Therefore, in accordance with [2], we can write

$$\| y\ (t) \| = \min_{u_i} \left\| \sum_{i=1}^{m} b_i u_i + A\ (t)\ \psi - \dot{\psi}\ (t) \right\|$$

and the function $\| y(t) \|$ cannot be further minimized by means of choice of matrix C. But the fundamental matrix of system (2.7) does depend on matrix C, and the latter must be chosen within the limits imposed upon it such that the deviation $z(t)$ is small.

This can be attained by imposing on the choice of matrix C the condition of improvement in some sense or other of the solutions of the homogeneous system (2.7). This example can be regarded as an automatic control system. In fact, if $\psi(t) \equiv 0$, system (2.7) is equivalent to the system

$$\frac{dx}{dt} = Ax + \sum_{i=1}^{m} ((C - A)\ x,\ b_i)\ b_i \tag{2.8}$$

The functions $u_i(x) = ((C - A)x,\ b_i)$ will be control functions in which the elements of matrix C occur as parameters. In the case matrices A and C are constants, there exist methods of selecting the parameters in such a manner (here the parameters are the elements of matrix C) that in some sense or other they allow us to improve the transient response of the automatic control system (2.8). Thus, for example, we can take advantage of the ideas of [6,7]. We can also choose the elements of matrix C in this computation such that the degree of stability [8] of system (2.7) is increased.

Remark. In Equation (2.8) let $m = 1$, i.e. we have the equation $u(x) = ((C - A)x, b)$, where $b(b_1, \ldots, b_n)$ is an n-dimensional vector. It turns out that the elements of matrix C can always be chosen such that by virtue of (2.8) with these controls the integral [9]

$$J(u) = \int_0^\infty \left(\sum_{k=1}^{n} a_k x_k^2 + cu^2 \right) dt$$

is minimized.

In fact, in [9] the equation of such a control was found in the form $u = p_1 x_1 + \ldots + p_n x_n$. In view of the arbitrariness of the variables $x_1, \ldots, x_n$, the equality $((C - A)x, b) = p_1 x_1 + \ldots + p_n x_n$ reduces to a system of nth order equations with n^2 unknowns c_{ik} which will be the elements of matrix C. Since $b_1^2 + \ldots + b_n^2 \neq 0$, it is not difficult to see that the obtained system of linear equations can always be solved with respect to c_{ik}.

Example. Let us consider the equation $\ddot{x} + a\dot{x} + bx = u$, which is equivalent to the following system of equations

$$\dot{x} = y, \qquad \dot{y} = -bx - ay + u \tag{2.9}$$

With respect to curve $\psi(t)$ let us assume that it degenerates to the origin of coordinates. Then, if the family of transient curves is given as the solution of the system of equations

$$\dot{x} = c_{11}x + c_{12}y, \qquad \dot{y} = c_{21}x + c_{22}y$$

the control has the form

$$u(x, y) = (c_{21} + b)x + (c_{22} + a)y$$

By substituting this control into system (2.9) we obtain

$$\dot{x} = y, \qquad \dot{y} = c_{21}x + c_{22}y$$

We shall choose the parameters c_{21}, c_{22} so as to increase the degree of stability. It is clear that on matrix C (see expressions (2.2), (2.7), (2.8)) some relations should be imposed associated with restrictions placed on the controls, and to the fact that systems (2.7) and (2.8) must be asymptotically stable at the origin. But these restrictions cannot eliminate the choice of matrix A for C, since for such a choice the control function in system (2.8) will be identically zero. In our example we shall consider that parameters c_{21} and c_{22} are bounded in modulus by the number N, where $N > \max\{1, |a|, |b|\}$. By immediate calculation we convince ourselves that the highest degree of stability in the given system is attained when $c_{21} = -N$, $c_{22} = -N$, if $N \leqslant 4$, and when $c_{21} = -1$, $c_{22} = -2\sqrt{N}$, if $N > 4$.

3. For simplicity in the subsequent reasoning we assume that matrices A and C are constant. Formula (2.6) then becomes

$$z(t) = F(t)\, z_0 + \int_{t_0}^{t} F(t-\tau)\, y(\tau)\, d\tau \tag{3.1}$$

By using this formula it is not difficult to obtain an estimate for the deviation $z(t)$:

$$\| z(t) \|_c \leqslant \max_i \sum_{k=1}^{n} | f_{ik}(t) | \, \| z_0 \|_c + \int_{t_0}^{t} \max_i \sum_{k=1}^{n} | f_{ik}(t-\tau) | \, \| y(\tau) \|_c \, d\tau \tag{3.2}$$

where $f_{ik}(t)$ are elements of the fundamental matrix $F(t)$, and $\| x \|_c$ in the given case is equivalent to $\max_i |x_i|$. From Expression (3.2) it is seen that the decrease in deviation $\| z(t) \|$ is connected with the decrease in the elements of the fundamental matrix of the solutions of system (2.7). To estimate the elements $f_{ik}(t)$ we make use of the results of [10]. Here, for convenience, we derive certain results of this paper in the necessary form.

Let there be considered a system of linear, homogeneous differential equations

$$dx \,/\, dt = Hx \tag{3.3}$$

We shall consider that matrix H does not depend on time. Let $G(x)$ be a positive-definite quadratic form, $g(x) = dG/dt$ by virtue of (3.3), and, finally, $N = \max g(x)$ when $G(x) = 1$. Then, for any coordinate of solution $x(t_0, x_0; t)$ of system (3.3), the following inequality

$$| x_s(t_0, x_0; t) |^2 \leqslant G(x_0) \frac{V_{n-1}^{(s)}}{V_n} e^{N(t-t_0)} \qquad (s = 1, \ldots, n;\ t_0 \leqslant t < \infty) \tag{3.4}$$

is satisfied.

Here V_n is the determinant of the matrix corresponding to quadratic form G, and $V_{n-1}^{(s)}$ is the minor of order $n-1$ obtained from determinant V_n by deleting the sth row and the sth column. We shall consider that matrix C is chosen such that the origin is asymptotically stable for system (2.7). As g let us choose the negative-definite quadratic form $g(x) = -(x, x) = -(x, Ix)$, where I is the identity matrix. Since system (2.7) is asymptotically stable with respect to the origin, we define the positive-definite quadratic form $G(x) = (x, Vx)$ such that $dG/dt = g$ by virtue of (2.7). The matrix V can be found [5, p. 429] from the matrix equation

$$H^{*}V + VH = I \qquad (H = A + D(C - A)) \tag{3.5}$$

In the given case

$$N = \max_{G(x)=1} [-(x, Ix)] = \max_{(x, Vx)=1} \frac{(x, -Ix)}{(x, Vx)} = \max \frac{(x, -Ix)}{(x, Vx)}$$

Further, it is not difficult to see [5, p. 257] that $N = -1/\mu$, where μ is the maximum eigenvalue of matrix V. By using Expression (3.4) for the elements of the fundamental matrix of the solutions of system (2.7), we can obtain the estimate

$$|f_{ik}(t)|^2 \leqslant G(e_k) \frac{V_{n-1}^{(i)}}{V_n} \exp\left(-\frac{1}{\mu}(t - t_0)\right) \tag{3.6}$$

where e_k is the unit coordinate vector. The expression occurring on the right-hand side of inequality (3.6) is a function of the elements of matrix C. By minimizing this function under some restriction on the elements of matrix C, we can achieve the reduction of the elements of the fundamental matrix of system (2.7), along with the reduction in deviation $\| z(t) \|$.

4. Now, on the right-hand sides of system (1.2) let there act persistent perturbations; then this system will have the following form

$$\frac{dx}{dt} = Ax + Bu + \varphi(t) \tag{4.1}$$

where the vector $\varphi(t) = (\varphi_1(t), \ldots, \varphi_n(t))$. The family of transient curves is given as the solution of the system of differential equations

$$df / dt = Cf \tag{4.2}$$

For simplicity let us consider that $\psi(t) \equiv 0$. For this case, by minimizing quantity (1.9), i.e. the quantity $\| Ax + Bu + \varphi(t) - Cx \|$, we find that

$$u_i(x, t) = ((C - A)x - \varphi, b_i) \qquad (i = 1, \ldots, m) \tag{4.3}$$

Let us now turn our attention to the case when the control function (4.3) can be computed by knowing the amount of perturbation only at given instants of time t. Without any difficulty whatsoever in the construction of the control functions, this permits us to consider systems subject to random perturbations. We shall divide every one of the control functions (4.3) into two parts $u_i^{(1)} + u_i^{(2)}$.

$$u_i^{(1)}(x,\ t) = ((C - A)x,\ b_i), \qquad u_i^{(2)}(t) = (-\varphi,\ b_i)$$

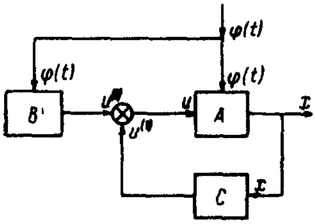

Fig. 2.

The function $u_i^{(1)}(x,\ t)$ will not depend on t if matrices A and C are constant. The block diagram which is described by Equation (4.1) is shown in Fig. 2. Here A is the object of control which realizes the required response, B is the corrective device, C is the feedback element. The perturbations $\varphi(t)$ enter into A.

At the same time they are fed into the corrective device B which selects the control action $u_i^{(2)}(t)$. The feedback element C selects the feedback signal $u_i^{(1)}(x,\ t)$. If it is required to realize some trajectory $\psi(t)$ using a system (4.1) subject to random disturbances, then the control action will have the following form

$$u_i^{(2)}(t) = \left(\frac{d\psi}{dt} - C\psi - \varphi,\ b_i\right)$$

and the feedback signal remains the same. As was shown in Section 2, the behavior of trajectory $\psi(t)$, and also the constantly acting perturbations $\varphi(t)$, do not influence the choice of matrix C which is connected only with system (1.2) itself and can be made beforehand in any fashion whatsoever.

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